

Chapter 1
The Investment Environment:
Markets & Securities

Capitalism

Modern capitalism is an economic system based on the mobility of money and financial capital. In short, market economies depend on people’s willingness to save a portion of their earnings which can then be invested in business enterprises. The process of moving savings into investment requires intermediation, financial intermediation. This transfer of personal savings to business investment is what creates economic growth. This is macroeconomics in a nutshell.

Business firms need money capital, i.e. cash, to acquire real capital, i.e. the means of production, to produce goods & services. Real Capital includes tangible assets such as offices, storefronts, warehouses, signage, computers, printers, copiers, office furniture, cars, trucks, supplies, inventory, etc. and intangible assets such as software, licenses, copyrights, patents, & trademarks, rights-of-way, plus standards & operating procedures, and additionally. We can also think of human capital as well. Human capital is typically created, not purchased and includes know-how, standards & practices, a trained & assembled workforce, etc.

Financial Capital & Financial Securities

To raise money capital, firms create financial capital in the form of financial securities. Financial securities are legal claims to future cash flows. Individuals and institutions exchange cash today for claims to future cash. Finance is the study of this inter-temporal allocation of cash between those who want to consume today and those who are willing (for a reward/premium) to consume later.

We can classify financial securities generically as either (a) fixed income securities, e.g. Bonds and (b) equity securities, e.g. Stocks. Firms create and sell stocks & bonds (financial capital) to acquire cash (money capital) in order to purchase the means of production (real capital).
Stocks & Bonds

Stocks originate in a private or in a public offering ("IPO") typically underwritten by an investment bank or two. Underwriting simply means that the bank(s) buy the shares from the firm and sell them to institutions and the public. Thereafter, the shares trade on secondary exchanges in financial markets.

The issuing firm receives cash from the investment banks only on the initial underwriting or, if additional shares are authorized, at secondary offerings.

Corporate bonds originate in a similar manner. Government bonds are issued by a government agency through an agent, sometimes an investment bank, sometimes electronically through a agent.

Stock and bond prices are reported daily. Stock and bond prices provide market-based information on the financial health of firms. Investors continuously analyze the financial performance of firms and watch security prices closely. Making thoughtful security purchases and selling securities in advance of poor firm performance is how investment managers try to “beat the market”.

Financial Markets

Financial markets are places where institutions and investors can buy and sell financial securities. Financial markets include both money markets and capital markets which are comprised of the exchanges or stock and bond markets, investment & commercial banks, and securities brokerages.

All banks are financial intermediaries. They intermediate between those who have money and those who need it. We can think of banks as institutions that rent
very large sums of cash in relatively small packages and then lease-out very large sums of cash is relatively large packages.

Firms raise short-term capital in the money markets and long-term capital in the capital the capital markets. Commercial banks are the primary financial intermediary in the short-term capital markets while investment banks are the primary middlemen in long-term capital markets. Each facilitate transactions between firms and investors for lines-of-credit to finance working capital, make loans, and underwrite the sale of shares of stock (the “IPO”) and bonds.

There are also institutions in adjunct financial markets including commodities markets, futures markets, foreign exchange markets, options markets, and insurance markets. Together, these markets facilitate the exchange of many types of financial securities each representing claims to future cash flows so that investors can spread the risk of financing new and existing business firms and commercial projects.

**Wall Street**

Wall Street was one of the early, and now the best organized, capital & financial markets. In addition to New York, we have well-organized financial markets in London, Tokyo, Hong Kong, Shanghai, Singapore, and Dubai.

When functioning properly, financial markets provide *liquidity* for firms and investors. Liquidity describes a market characteristic of an asset or a financial security. It means “quick & easy to sell at a fair price”. This is the nature, purpose, and the advantage of markets in general – a place to make transactions quickly and fairly.

**World Capital Markets**

As of 2011, global capital was estimated at $212 trillion with stocks about $54 trillion and bonds $158 trillion.¹ In the United States, at 2012, the stock market was $21 trillion and the bond market $37 trillion.² Thus, the U.S. stock market is 40-45 percent of global equity capital while U.S. bonds comprise 20-25 percent of global debt capital. Approximately $1 trillion of global equity capital represents *Emerging Markets.*

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² Bloomberg.
The four largest emerging markets are the BRIC countries - Brazil, Russia, India, and China. The next six emerging market five countries are South Korea, Mexico, Indonesia, Turkey, Saudi Arabia, and Iran.\(^3\)

- PIMCO’s world bond fund, PSAIX.
- Vanguard’s world equity fund VT.
- Morgan Stanley’s Capital International All country World Index, MSCI ACWI.

Valuation
The value of a financial security is the present value of expected future cash flows discounted at an appropriate risk-adjusted discount rate (“RADR”). The adjectives expected and appropriate are especially germane. Future cash flows carry some degree of uncertainty and discount rates need to be relevant to both the source of the cash flows (the issuer) and competing alternatives (other similar securities). An assessment of the risk, i.e. the possible variation in future cash flows, is particularly important because this will determine the risk premium which investors require for bearing risk. A risk premium is simply a reward for bearing risk.

Future cash flows from financial securities include:

1) Interest payments, called Coupons, on Bonds;
2) Dividend payments from Shares of Stock;
3) Return of principal, the Face, from a Bond;
4) Capital gains, Stock Price appreciation;

The Investing Process
Investing is how we make money work for us. There are four steps, at various points, in the investment process:

- Asset Allocation
- Risk Tolerance
- Management Style: active versus passive
- Security Selection

Asset allocation is the process of deciding what proportion of our savings will be invested in the different types of financial securities. To simplify this, we typically

\(^3\) Wikipedia.
think of two kinds of assets – fixed income assets, such as money market securities, bonds, and real estate - and equity assets, e.g. common stocks and derivatives.

a) **Risk tolerance** means the level of uncertainty that the investor is willing to bear understanding that the empirical record demonstrates an inverse relation between risk and reward, call the *risk-return trade-off*.

b) **Management style** is the preference for a combination of “picking securities individually” or investing in a broad portfolio of pooled securities.

c) **Security selection** is the process of choosing specific securities for the “active” investor. There are twin goals for the active manager:

1) Finding undervalued securities
2) Timing the market, i.e. *buying low and selling high*.

**Indexes and Index Funds**

Investing by searching for individual securities is called Active Investment Management. Alternatively, investors can invest in collections of securities, called *Funds*, usually managed by *fiduciary*-minded professionals. This is Passive Investment Management on the part of the individual investor. – Fund managers may be active, selecting individual securities for the subject fund, or passive if the fund in an *Indexed Fund*.

Indexed funds are composed of portions of all of the securities in an *index*. An index is merely a stylized, formal way of tracking the composite prices of all of the securities in a selected class or collection of securities. Collections of securities that might be indexed include:

a) Selected industries – communication, bio-technology, transportation, etc.

b) Selected geography – Far East, Brasil, Turkey, etc.

c) Company size – the DOW Thirty, Fortune 100, S&P 500, Russell 2000, etc.

that there is information in index movements; others believe there is more emotion than information.\textsuperscript{4}

The most reported index is the Dow-Jones 30 Industrials. Below is a chart of the DOW for five years to the end of June 2015. During this five-year period, the index moved from 9,686 to a peak of 18,272 on May 15, 2015.

Source: Google.

\textsuperscript{4} John Maynard Keynes used the term “animal spirits”.
## The Dow Jones Industrial Average Companies since March 18, 2015

<table>
<thead>
<tr>
<th>Company</th>
<th>Symbol</th>
<th>Industry</th>
<th>Added</th>
</tr>
</thead>
<tbody>
<tr>
<td>3M</td>
<td>MMM</td>
<td>Conglomerate</td>
<td>1976</td>
</tr>
<tr>
<td>American Express</td>
<td>AXP</td>
<td>Consumer finance</td>
<td>1982</td>
</tr>
<tr>
<td>Apple</td>
<td>AAPL</td>
<td>Consumer electronics</td>
<td>2015</td>
</tr>
<tr>
<td>Boeing</td>
<td>BA</td>
<td>Aerospace and defense</td>
<td>1987</td>
</tr>
<tr>
<td>Caterpillar</td>
<td>CAT</td>
<td>Construction and mining equipment</td>
<td>1991</td>
</tr>
<tr>
<td>Chevron</td>
<td>CVX</td>
<td>Oil &amp; gas</td>
<td>2008</td>
</tr>
<tr>
<td>Cisco Systems</td>
<td>CSCQ</td>
<td>Computer networking</td>
<td>2009</td>
</tr>
<tr>
<td>Coca-Cola</td>
<td>KO</td>
<td>Beverages</td>
<td>1987</td>
</tr>
<tr>
<td>DuPont</td>
<td>DD</td>
<td>Chemical industry</td>
<td>1935</td>
</tr>
<tr>
<td>ExxonMobil</td>
<td>XOM</td>
<td>Oil &amp; gas</td>
<td>1928</td>
</tr>
<tr>
<td>General Electric</td>
<td>GE</td>
<td>Conglomerate</td>
<td>1907</td>
</tr>
<tr>
<td>Goldman Sachs</td>
<td>GS</td>
<td>Banking, Financial services</td>
<td>2013</td>
</tr>
<tr>
<td>The Home Depot</td>
<td>HD</td>
<td>Home improvement retail</td>
<td>1999</td>
</tr>
<tr>
<td>Intel</td>
<td>INTC</td>
<td>Semiconductors</td>
<td>1999</td>
</tr>
<tr>
<td>IBM</td>
<td>IBM</td>
<td>Computers and technology</td>
<td>1979</td>
</tr>
<tr>
<td>Johnson &amp; Johnson</td>
<td>JNJ</td>
<td>Pharmaceuticals</td>
<td>1997</td>
</tr>
<tr>
<td>JPMorgan Chase</td>
<td>JPM</td>
<td>Banking</td>
<td>1991</td>
</tr>
<tr>
<td>McDonald's</td>
<td>MCD</td>
<td>Fast food</td>
<td>1985</td>
</tr>
<tr>
<td>Merck</td>
<td>MRK</td>
<td>Pharmaceuticals</td>
<td>1979</td>
</tr>
<tr>
<td>Microsoft</td>
<td>MSFT</td>
<td>Consumer electronics</td>
<td>1999</td>
</tr>
<tr>
<td>Nike</td>
<td>NKE</td>
<td>Apparel</td>
<td>2013</td>
</tr>
<tr>
<td>Pfizer</td>
<td>PFE</td>
<td>Pharmaceuticals</td>
<td>2004</td>
</tr>
<tr>
<td>Procter &amp; Gamble</td>
<td>PG</td>
<td>Consumer goods</td>
<td>1932</td>
</tr>
<tr>
<td>Travelers</td>
<td>TRV</td>
<td>Insurance</td>
<td>2009</td>
</tr>
<tr>
<td>UnitedHealth Group</td>
<td>UNH</td>
<td>Managed health care</td>
<td>2012</td>
</tr>
<tr>
<td>United Technologies</td>
<td>UTX</td>
<td>Conglomerate</td>
<td>1939</td>
</tr>
<tr>
<td>Verizon</td>
<td>VZ</td>
<td>Telecommunication</td>
<td>2004</td>
</tr>
<tr>
<td>Visa</td>
<td>V</td>
<td>Consumer banking</td>
<td>2013</td>
</tr>
<tr>
<td>Wal-Mart</td>
<td>WMT</td>
<td>Retail</td>
<td>1997</td>
</tr>
<tr>
<td>Walt Disney</td>
<td>DIS</td>
<td>Broadcasting and entertainment</td>
<td>1991</td>
</tr>
</tbody>
</table>
Chapter 2
Returns to Investing

Investment Returns

Purchasers of financial securities expect both a *return “of”* and a *return on* the initial investment. Total returns, however, are not always positive. Investors don’t know for certain how well the firm and the investment in the firm will perform. This uncertainty means that financial securities are *risky.*

Below is a list of return formulas which we will use over-and-over again. Unfortunately, many similar formulas have different names. We will get used to this. More or less all return formulas are simply rearrangements of the fundamental compounding equation:

**Equation [1-1]** \[ FV = PV \left(1 + r\right)^t \]

\( r \) is an annual rate-of-return, a rate-of-interest, so to speak, and \( t \) is the number of years it takes to go from PV ("present value") to FV ("future value").

We can rearrange Equation 1-1 for "\( r \)"

**Equation [1-2]**

\[ r = \left(\frac{FV}{PV}\right) \exp\left(\frac{1}{t}\right) \]

This will be the most useful and most used equation in this course because it portrays the fundamental investment objective, i.e.

a) buying *the highest* \( FV \)

b) for *the lowest* \( PV \)

c) over *the shortest time* "\( t\)"

d) in order to earn *the highest return* "\( r\)"!

To use this formula effectively:

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1 See the appendix for the mathematics of the Time Value of Money.
• substitute \( P_0 \) or \( I_0 \) - meaning opening price or initial investment for \( PV \).

• substitute \( P_T \) or \( I_T \) - meaning closing price or ending investment value for \( FV \).

Here is a list of related “return” formulations that we will fill-in and calculate using market data.

\[
\begin{align*}
\text{TOTAL RETURN} & \quad \left( \frac{FV}{PV} \right) \\
\text{HPR} & \quad \left( \frac{P_T}{P_0} \right) - 1 \\
\text{AHPR} & \quad \left( \frac{P_T}{P_0} \right)^{1/t} - 1 \\
\text{CAGR} & \quad \left( \frac{P_T}{P_0} \right)^{1/t} - 1 \\
\text{APR} & \quad \left( \frac{P_T}{P_0} \right)^{1/t} - 1 \\
\text{APY} & \quad \left( 1 + \frac{APR}{n} \right)^{nt} - 1
\end{align*}
\]

We can compare this large cap index to a single mid-cap security for the same period. Nordstrom’s stock moved from $32.17 to a peak of $82.32 on March 20, 2015.
Calculate Some Returns and Compare

Nordstrom stock hit low of $6.61 on November 21, 2008. It reached a recent high of $82.32 on March 20, 2015. Given that information, calculate the Total Return, the HPR, and the CAGR (on the capital gain) for an investor who might have been prescient enough to have acquired a few shares on 11/21/2008, say 10,000 shares, and sold them on 3/20/2015. Be as precise as you can. In addition, test your research capabilities by also calculating the total amount of dividend income that this investor received during that period. Choose any one of the DOW 30 Industrial stocks listed below and do the comparable calculation. Comments?
Chapter 3

Fixed Income Securities

Fixed income securities are debt securities. They include money market instruments, corporate and government bonds. All fixed income securities are basically loans, i.e. the investor is the lender and the security issuer, firms, institutions, or governments are the borrowers.

The Bond Market

The U.S. Government is the largest issuer of bonds in the world. In 2008, there was over $3.5 Trillion in outstanding U.S. government debt. The bond market exceeds $158 Trillion.

Bonds sell in an auction environment with buyer bidding a price to yield. In other words, a price that will, all thing equal and reliable, give the lender a specified return. Of course, all things seldom remain equal, so bond price bids offer information as to what lenders think about the future. This information bi-product is one of the important functions of capital markets. In addition, the “yield” is an interest rate. Thus, bond markets are the source of interest rates. In fact, since the U.S. Government is the most credit-worthy borrower in the world, it pays the lowest interest rate on its borrowing. Therefore, investors regard the yield on the U.S. 10 Year bond, the benchmark yield, i.e. the lowest reference point for worldwide interest rates.

The Bond

All loans involve an exchange of cash from lender to borrower. The borrower receives a lump sum of cash today in exchange for future payments. This means that the investor is essentially buying a series of future cash flows. These future cash flows repay the amount loaned, the principal, plus interest. The future cash flows repay the principal and interest in one of several ways:

a) In a single lump sum at a specified future date;
b) In a series of fixed, periodic, future payments; or
c) In a series of fixed periodic, future payments plus one or two future lump sum payments, called balloons.

1 Real estate is considered a fixed income asset but not a fixed income security, per se.
The point is that the initial exchange, the principal amount loaned, is a present value so the present value of the future cash flows must return a present value equivalent to the present value of the principal borrowed. Thus:

\[
\text{The Amount Loaned} = \text{PV(loan payments)}
\]

When a loan is repaid by fixed, periodic payments (annuity style), we say that the loan is fully amortized by the payments. In other words, each payment includes interest and a portion of principal. The present value of all of the payments is exactly equal to the original principal borrowed.

A bond is a loan made by the bond’s initial investor to the issuing firm. As with any financial security, the bond’s Price is the present value of the future cash flows

The borrowing entity, typically a firm, sells the lender, an individual or institution, one or both of two types of future cash flows:

1) The Coupon (“C”); this is the periodic cash flow
2) The term (“T”); when the investor receives the last coupon and
3) The Face of the bond, typically $1,000 or maybe $10,000.

The Coupon is equal to the coupon rate (“CR”) x the Face (“F”). For example, a 10 year, $1,000 Face bond with a 10% coupon will pay $100 per year for ten years plus 1,000 in ten years.

**Bond Valuation & Pricing**

The coupon rate is not an interest rate. The bond’s term (“T”) is called the bond’s maturity, which in this example is 10 years

In exchange for buying the bond, the investor can expect the $Coupons and the $Face. These are basically the loan payments which are compensation for lending plus the Principal. This is much like compensation from a non-amortizing loan.

Unlike conventional loans which have one lender and one borrower, bonds may have multiple lenders because bonds sell – lenders sell to lenders – in the bond market. The debt of the borrower is re-priced with each transaction. The amount that the new investor pays the old investor sets interest rates for similar loans. The price received by the selling investor or the bond determines several measures of investor success, or failure:
• The Total Return (“TR”):
  Total CF’s through “T” divided by initial outlay (“I₀”) minus 1.

• Annualized Holding Period Return (“AHPR”):
  The TR ^ (1/T) minus 1. This is the annual “r” that, when
  compounded”, exactly connects the initial outlay I₀ with the Total
  Future CFs. The AHPR is the same as a CAGR.

We can conclude that:
  a) The less the bond investor pays for the bond, the higher the return to the
     investor because the bond’s issuer is paying more in total interest;
  b) The more the bond investor pays for the bond, the lower the return to the
     investor because the bond’s issuer is paying less in total interest.

Thus, bond prices and interest rates are inversely related. This inverse relationship is
not linear, as we shall see later.

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     investor because the bond’s issuer is paying less in total interest.

Thus, bond prices and interest rates are inversely related. This inverse relationship is
not linear, as we shall see later.

To calculate the bond’s price we discount the expected future cash flows by a
risk-adjusted discount rate (“RADR”). This is the (at-the-moment) appropriate market
rate of interest for the term of the cash flows and the borrower's credit-rating on that particular bond. Consider the following example:

We have a 10 year, 10 percent coupon bond. Assuming the appropriate RADR is 8 percent, which is different from the coupon rate, we can calculate the present value (bid price) of this bond:

\[ C \times PVFA(r, t) + F \times PVF(r, t) \]

\[ C \times PVFA(8\%, 10) + F \times PVF(8\%, 10) \]

\[ 100 \times PVFA(8\%, 10) + 1,000 \times PVF(8\%, 10) \]

\[ 100 \times \left[ \frac{1 - (1+r)^{-t}}{r} \right] + 1,000 \times \left[ \frac{1}{(1+r)^t} \right] \]

Finish this calculation:

The relationship between the Coupon rate ("CR") and market interest rates (the basis for the discount rate "DR") will determine whether the subject Bond is priced at a discount or a premium to the bond’s Face, also called the bond’s par value. Teasing-this-out, we can conclude that a bond’s price will be:

a) **below Face** if the CR < DR and we have a discount bond

b) **equal to Face** if the CR = DR and we have a par bond

c) **greater than Face** if the CR > DR and we have a premium bond
For example, consider a 5 year 5 percent coupon bond. Calculate the price of this bond if interest rates are 4%, 5%, and 6%:

\[ PVFA (4\%, 5) \times 50 \text{ plus } PVF (4\%, 5) \times 1,000 = \]

Risk-Adjusted Discount Rates ("RADR’s")
The appropriate discount rate for valuation is influenced by many factors. For example, RADR’s vary with/by:

- the term of the security, thus the term structure of interest rates, called the Yield Curve
- the market – government, corporate, and municipal securities
- the bond’s unique characteristics, called covenants
- the borrower's characteristics, called credit risks
- inflation

The most important distinction amongst discount rates is the relative, general level of interest rates represented by the U.S. Treasury Yield Curve.

The U.S. Treasury yield curve is often used as a baseline for determining the interest rate appropriate for any given maturity. Yields ("prevailing interest rates") can be found at:


Research
Using the link above, insert the current Yield Curve and the yield curve for one, five, and ten years go.

<table>
<thead>
<tr>
<th>Date</th>
<th>90-day</th>
<th>1-year</th>
<th>2-years</th>
<th>5-years</th>
<th>10-years</th>
<th>30-years</th>
</tr>
</thead>
<tbody>
<tr>
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</tbody>
</table>
We have *working names* for two of the U.S. Treasury yields:
1. The 10-year treasury yield is called the benchmark yield, and
2. The 3-Month Treasury yield is called the risk-free rate ("r_f").

It is important to understand that bonds of the same maturity may have different yields for a couple of reasons:
   a) Bonds might have different issuers – a *credit effect* – and/or
   b) Bonds may have different cash flows – a *coupon effect*.

Thus, we can reduce the list above for the three primary factors that influence the choice of a discount rate:
- Credit quality
- Time
- Inflation

We can think of the composite discount rate as having a marginal component representing the risk premium required by investors related to each of these factors. Representative premiums are illustrated in the chart below (as of June 2015):

<table>
<thead>
<tr>
<th>Bond Type</th>
<th>Yield</th>
<th>Premium</th>
</tr>
</thead>
<tbody>
<tr>
<td>10-year BBB corporate bond</td>
<td>3.89%</td>
<td>156 bps</td>
</tr>
<tr>
<td>10-year U.S. Treasuries</td>
<td>2.33%</td>
<td>225 bps</td>
</tr>
<tr>
<td>3-month T-Bills</td>
<td>8 bps</td>
<td>(38 bps)</td>
</tr>
<tr>
<td>10-year Real Interest Rate</td>
<td>46 bps</td>
<td></td>
</tr>
</tbody>
</table>

The real rate of interest is determined by macro-economic variables such as the cost of investment capital and the productivity of real capital as well as the demand for, and supply, of savings.
Inflation is a sustained rise in the general level prices. Investors demand an inflation premium as compensation for the loss of purchasing power as they wait for cash flows. The 90-day T-Bill rate is used as a proxy for the nominal rate of interest and the real rate of interest is a construct calculated by subtracting the expected rate of inflation from the T-Bill rate. Currently, the real rate of interest appears to be negative.

The quality premium reflects the credit-riskiness of the borrower/issuer. U.S. Treasury securities carry no perceived credit risk. At the onset of a recession, the yield spread between U.S. Treasuries and corporate securities widens due to a flight-to-quality.

Discount Bonds
A discount bond, also known as (“aka”) a zero-coupon bond, has a coupon rate of 0%. The only cash flow from a discount bond is its terminal cash flow, normally the face. The total return on discount bonds is:

\[
\frac{\text{Terminal CF}}{\text{Price Paid}} - 1
\]

This measures return without adjustment for the time it took to collect the terminal cash flow. However, we can annualize the return, i.e. find the CAGR, using the following familiar formula:

\[
\left( \frac{\text{All Proceeds}}{\text{Initial Investment}} \right) \exp\left( \frac{1}{t} \right) - 1
\]

Example. Do the CAGR for a 20 year zero-coupon bond with a face value of $10,000 selling for $5,000.
Bond Pricing with Non-Flat Term Structure of Interest Rates

Previously, we discounted all cash flows at the same RADR which is the result of assuming a flat yield curve.

Treating cash flows one-year from now exactly the same as cash flows five or ten years from now is naïve, it contradicts empirical evidence. Time matters because time introduces additional risk. Thus, there is no reason to assume that the present value of all future dollars should be discounted by the same RADR. Therefore we turn to the yield curve for RADRs for different time periods.

For example, what if we assume that the term structure of rates is linear and perfectly correlated to time.

<table>
<thead>
<tr>
<th>Year</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rate</td>
<td>1 %</td>
<td>2 %</td>
<td>3 %</td>
<td>4 %</td>
<td>5 %</td>
</tr>
</tbody>
</table>

What is the present value of a 5-year, 5 percent coupon bond?

<table>
<thead>
<tr>
<th>Year</th>
<th></th>
<th></th>
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<tr>
<td>Present Value</td>
<td></td>
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</table>

Notice that, when the term structure is not flat, the price and the yield, RoR will depend on the holding period and we can calculate a unique RoR called the yield-to-maturity ("YTM"). The YTM is a single discount rate that will set the net present value of the bond purchase to zero.

The YTM is a good proxy for a RADR. It reflects the price of time for uniformly structured cash flows, characteristic of coupon bonds.

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Rolling Down the Yield Curve

This is a nice money-making trick. Don’t ask me why more individual investors don’t do it, maybe they do but don’t brag about it. It is not intuitively obvious, but the mathematics prove the argument.

This strategy requires a yield curve that is positively sloped, the steeper the better. It involves buying a 4 to 5 year, or longer, bond and selling it after 2 to 3 years. The strategy produces a higher yield than holding the bond to maturity.

As a reference point, let’s price a 5-year, 5% coupon with a flat yield curve at 5 percent. Since the discount rate and the coupon rate are equal, we have a bond priced at par, $1,000. This bond’s YTM is 5 percent.

Next, consider the same sloped yield curve we used earlier. Here it is:

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<thead>
<tr>
<th>Year</th>
<th>Rate</th>
<th>CF</th>
<th>PVF</th>
<th>PV</th>
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<tr>
<td>4</td>
<td>4 %</td>
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<tr>
<td>5</td>
<td>5 %</td>
<td>$1,050</td>
<td>0.78</td>
<td>$822.70</td>
</tr>
</tbody>
</table>

This bond’s YTM is 4.642 percent.

Using this as our point of reference, let’s imagine that interest rates do not change, i.e. the yield curve remains as it was, and we sell this bond one year later, just after we receive the 1st $50 coupon. What price would we receive? The numbers are below:
Next year, our bond will be a 4-year, 5% coupon bond. It sells for more than we paid for it because the terminal cash flow, twenty times larger than the others, is one-year closer and discounted at 4% rather than 55. Thus, it is worth:

\[
\begin{align*}
\text{PV} &= 897.54 \\
\text{(\$822.70)} \\
\text{\$74.84} &> \text{more than we paid for it. Plus, we received a \$50 coupon payment.}
\end{align*}
\]

Our proceeds are \$1,090.86. We invested \$1,008.76 a year earlier, so our annualized RoR is:

\[
\left( \frac{\text{\$1,091}}{\text{\$1,009}} \right) - 1 = 8.12\% 
\]

**The Characteristics of this Strategy**

This yield boost is due to several characteristics of bond investing:

1) Longer term bonds pay more interest than shorter term bonds
2) Longer term bonds rise in value over time relative to shorter term bonds which are less risky so investors will pay more for them
3) As a bond moves closer and closer to its maturity date, its yield moves closer and closer to zero
4) This strategy works for premium bonds, just not as well as with discount bonds
5) This works better the steeper the yield curve, thus better for corporate bonds as long as default risk is not a factor, but
6) This strategy worsens if overall rates rise during its execution.
Time Value of Money Mathematics

Notation

- \( r \) is the annual discount
- \( t \) is the number of years
- \( n \) is the number of compounding periods per year

For single sums of money between points in time, we start with the basic, annual compounding equation:

\[
[1] \quad FV = PV \left(1 + r\right)^t \quad \text{we call} \quad (1 + r)^t \quad \text{the FVF (“future value factor”)}
\]

Transform [1] into the annual discounting equation:

\[
[2] \quad PV = FV \left(1 + r\right)^{-t} = FV \frac{1}{(1+r)^t}
\]

\((1 + r)^t\)

and

\[\frac{1}{(1 + r)^t}\]

... are unit-neutral factors, the Future Value Factor (“FVF”) and the Present Value Factor (“PVF”), respectively, for annual compounding. We might write them as:

- FVF\((r, t, n)\) and PVF\((r, t, n)\) where \(n=1\).

Should compounding or discounting be done for often than annually but for one-year or more years we would write that the FVF for 10 percent per year, over 10 years, compounded monthly, the FVF \((10\%, 10, 12)\) would be written:

\[
(1 + \frac{10\%}{12})^{120}
\]
Now, consider a series of payments, “annuities”, where we want to know the future value of a series of fixed annual payments (“$A$”). We say that the future value of an annuity $A$ for $t$ years at $r$ percent is:

\[ FVA = \sum_{t=1}^{T} (1 + r)^t A_t = A \sum_{t=1}^{T} (1 + r)^t \]

Where

\[ \sum_{t=1}^{T} (1 + r)^t \] is abbreviated the FVFA ($r$, $t$, $n$) which simplifies to

\[ \frac{(1+r)^t - 1}{r} = \frac{1}{r} ((1 + r)^t - 1) \]  

Conversely, we want to know the present value of series of fixed annual receipts (“$A$”). We say that the present value of an annuity $A$ for $T$ years at $r$ percent is:

\[ PVA = PVA = \sum_{t=1}^{T} \left( \frac{1}{1+r} \right)^t A_t = A \sum_{t=1}^{T} \left( \frac{1}{1+r} \right)^t \]

Where

\[ \sum_{t=1}^{T} \left( \frac{1}{1+r} \right)^t \] is abbreviated the PVFA($r$, $t$, $n$) which simplifies to:

\[ \frac{1 - (1+r)^{-t}}{r} = \frac{1}{r} \left[ 1 - \frac{1}{(1+r)^t} \right] \]
Since the four factors – FVF, PVF, FVFA, and PVFA – from the four equations above are each combinations of r, t, n, we can create Tables of them. These four Tables appear at the back of this Appendix. Each Table assumes that n = 1. If (r, t, n) is not = 1, then divide r by n and multiply t x n.

Let’s try a few:
Future Value of $1 Table of *Future Value Factors* ("FVF") = \((1 + r)^t\)

<table>
<thead>
<tr>
<th>Periods</th>
<th>(1.00%)</th>
<th>(2.00%)</th>
<th>(3.00%)</th>
<th>(4.00%)</th>
<th>(5.00%)</th>
<th>(6.00%)</th>
<th>(7.00%)</th>
<th>(8.00%)</th>
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<th>(10.00%)</th>
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Table 1 FVFs
### Present Value of $1 Table of Present Value Factors ("PVF") = \((1 + r)^{-t}\)

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Table 2 PVFs
FV of $1 "ordinary" Annuity Table: ("FVFA") = \[(1 + r)^t - 1\] \times \frac{1}{r}

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<th>2.00%</th>
<th>3.00%</th>
<th>4.00%</th>
<th>5.00%</th>
<th>6.00%</th>
<th>7.00%</th>
<th>8.00%</th>
<th>9.00%</th>
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Table 3 FVFAs
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<th>3.00%</th>
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<th>5.00%</th>
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<th>7.00%</th>
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Table 4 PVFAs
Practical Problem #1

Each of us faces a generic economic life cycle where, in general, we first consume, then we save & consume, and lastly we consume. These phases are roughly correlated with our early life (as children), our adult years (working), and our retirement years. To be financially secure, we must save enough during the middle “working” phase to finance spending in retirement. Divide the process into three phases: (1) Saving, (2) Investing, and (3) Spending.

i. Starting at age 25, save and invest on an annual annuity basis;

ii. From age 45 to 65, no additional saving, just invest the accumulation from age 25 to 45;

iii. In retirement, age 65 to 85, spend on an annual annuity basis.

If you believe that you need $100,000/year to be comfortable in retirement, then without investing you will need $2M in savings at age 65. To achieve this while working and not investing, you will need to save $50,000 per year. This is a daunting task, especially in the presence of taxes, to say nothing of children and bad habits like sleeping in a bed and eating hot food a few times a day.

Today, Defined Benefit Plans (“DBP”), financed by employers, are a progressively rare manner of retirement funds. Instead, most individuals will rely on Defined Contribution Plans (“DCP”) which are self-financed such as the 401k, 403b, Roth and SEP IRA’s.

The question is how much does one need to save, for how long, and earn what rate-of-return to fund retirement spending, – financed by employers  Even a modest annual rate of return (“RoR”) can reduce the required savings necessary to finance a modest retirement annuity if individuals start early.

Here is an exercise to examine the effect that an investment returns can have on the retirement saving problem. Imagine that you want to spend $100,000 per year from age 65 to 85 and that you don’t believe that you will be able to save anything from age 45 to 65, so all of your savings need to be made from age 25 to 45. You believe that reasonable expected annual rates-of-return are as follows:
Age 25-45 8 percent
Age 45-65 6 percent
Age 65-85 4 percent

How much must you save and invest from age 25 to 45 so that you can spend $100,000 per year from age 65 to 85 assuming zero savings for the 20 years from age 45 to 65 but investing what accumulated from age 25 to 45.

It is easier if we sketch this problem in its three phases listing the parameters – amounts and rates. Before we solving it, write down your best guess as to how much you think you will need to save per year for those initial 20 years in order to spend $100,000 per year for the last 20 years:

Save $____________ per year from age 25 to 45. OK. Now let’s do the calculation.
Practical Problem #2

You want to buy a Tesla S4. Assume that this car’s cost, including options, fees, and taxes is $100,000. Calculate the monthly loan payments on a $100,000 loan over 6-years at 5 percent. You are borrowing $100,000 in present value. You plan to repay this present value with 72 future, monthly payments. Thus, the present value, at 5 percent, of these 72 future, monthly payments must equal $100,000.

$$PV(PAYMENTS) = \$100,000 = \text{PAYMENT (“$A”) \times PVFA(r=5\%, \ t=6, \ n=1)}$$

Calculated on monthly, not an annual, basis.

Let’s start by looking at the annual compounding PVFA:

$$PVFA|_{T, \ r} = \frac{1-(1+r)^{-t}}{r} = \frac{1}{r} \left[1 - \frac{1}{(1+r)^t}\right]$$

... and since we want the monthly PVFA, we make some adjustments to our parameters. We have $T \times 12 = 72$ periods and must apply only $12^{th}$ the annual rate as the discount rate:

$$\frac{5\%}{12} = 0.00417 = 0.417\% \text{ or } 47.1 \text{ basis points} \text{ per month, so the PVFA calculation, in detail, is:}$$

$$PVFA = \frac{1-(1+\frac{5\%}{12})^{-72}}{\frac{5\%}{12}} = \frac{1}{0.00417} \left[1 - \frac{1}{(1.0041700)^{72}}\right]$$

$$PVFA = \frac{0.2587}{0.00417} = 62.043$$

Returning to the payment calculation:

$$\$100,000 = 62.043 \times \$A, \text{ and}$$

$$\$A = \$1,611.78 \text{ per month for 72 months.}$$
Total dollars paid for the car will be $116,048.16, comprised of $100,000 in loan principal plus $16,048 in interest. The present value of the loan payments is exactly equal to $100,000. The loan payments *fully amortize* the loan -meaning that the payments “kill-off the amount owed including interest”. 
Practical Problem #3

A client hires you to invest $10,000 for five years. The client will tolerate the least risk for a 5-year investment. Assume a flat yield curve at 5 percent.

On behalf of this client, you purchase ten five-year, 5-percent coupon bonds at Par.

The next day, interest rates rise 500 basis points across the term structure.

a) What happens to the market value of the client’s bonds?

b) Respond to the client’s husband’s complaint that “you” caused his family to lose a substantial amount of money overnight?

c) What could the client do if she fired you and reinvested the money in a comparable security?