New Scenario

We can trade a single sum of money today, a (PV) in return for a series of periodic future payments (FV’s).

This is what a Loan is …

• Borrow (Lend) today a large single amount) and make (payments) or receive payments in the future to repay the Loan or recover the Loan.
Annuities, again

• An annuity is a “fixed” periodic payment or deposit:
• These payments that are made at the end of the financing period are called “Ordinary” Annuities.
• This is the only type of payment that we will consider in this section of the course.
• If you borrow (take a mortgage), you agree to pay an Ordinary Annuity because your 1st payment is not due the day you borrow, but one month later.
The PV of an Annuity

In the Slides TVM 2a, we calculated the FV of $10,000/year for 5 years at 5 percent and found that that future value was $55,256, i.e. we calculated:

$A \times FVFA(r, t) = FV$

$10,000 \times \left[ \frac{\left[1-(1+r)^t\right] - 1}{r} \right] = FV$

$10,000 \times 5.5256 = 55,256$

Let’s reverse this process by asking - what is the PV of $10,000/year for 5 years at 5 percent?

It will be < $50,000 because each of the five $10,000 installments are paid in the future, so they need to be discounted.
Calculate the PV of each $10,000 payment by using Table 2 and taking the PVF’s from the 5% column down to 5 years.

**Table 2 applied to the $10,000 installments.**

<table>
<thead>
<tr>
<th>Periods</th>
<th>Interest = r 5.00%</th>
<th>PVFs</th>
<th>Installments</th>
<th>Present Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.9524</td>
<td>$10,000</td>
<td>$9,524</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>0.9070</td>
<td>$10,000</td>
<td>$9,070</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>0.8638</td>
<td>$10,000</td>
<td>$8,638</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>0.8227</td>
<td>$10,000</td>
<td>$8,227</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>0.7835</td>
<td>$10,000</td>
<td>$7,835</td>
<td></td>
</tr>
<tr>
<td></td>
<td><strong>4.329</strong></td>
<td><strong>$43,295</strong></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
The PV of an Annuity

Rather than run the sum of five products, we can factor-out the $10,000 and sum the PVFs, then find the product.

\[ = 10,000 \times \left( \sum (1.05)^{-t} \right) \text{ for } t=1 \text{ to } 5 \]

\[ = 10,000 \times \text{PFVA} (r=5\%, \ t=5) \]

\[ = 10,000 \times 4.329 \text{ from Table 4} \]

\[ = 43,295 \]
Present Value Annuity Factors

Table 4 is constructed using this formula

Each Factor, called a PVFA, is defined by its rate “r” and its time in years “t”:

\[ PVFA(r, t) = \frac{[1 - (1+r)^{-t}]}{r} \]

These are called Present Value Factors of Annuities and are found on the PVFA Table 4.
Calculating the *Present Value Annuity Factor*

Table 4 is constructed using this formula

\[
PVFA(r, t) = \left[ 1 - PVF(r, t) \right] / r
\]

Each Factor, called a PVFA, is defined by its *rate* “r” and its *time* in years “t”:

For the previous example:

\[
PVFA(5\%, 5) = \left[ 1 - PVF(r, t) \right] / r
\]

\[
PVFA(5\%, 5) = \left[ 1 - (1.05)^{-5} \right] / 0.05
\]

\[
PVFA(5\%, 5) = \left[ 1 - 0.7835 \right] / 0.05
\]

\[
PVFA(5\%, 5) = \left[ 0.2165 \right] / 0.05
\]

\[
PVFA(5\%, 5) = 4.3295
\]
The PV of an Annuity

The present value of an annuity is the Annuity \times the PVFA(r, t).

In our first example it is:

\[ = \$10,000 \times PVFA(5\%,5) \]

\[ = \$10,000 \times 4.329 \text{ where we find 4.329 by calculating it or looking on Table 4} \]

\[ = \$43,295 \]
The PV of an Annuity in Reverse

In the prior example, we found that the PV of a 5 year $10,000 ordinary annuity at 5% is $43,295. Reversing that, let’s ask:

- What is the FV of a 5 year $10,000 ordinary annuity at 5%?

Formulate it:

\[
\text{FV(Annuity)} = \text{Annuity} \times \text{FVFA}(5\%, 5) \\
\text{FV(Annuity)} = $10,000 \times 5.5256 \\
\text{FV(A =$10,000)} = $55,526
\]
The PV of an Annuity in Reverse

What do we have now?
• We have a Present Value of $43,295.
• We have a Future Value of $55,256.
• These are linked by 5 years?
• What is the CAGR that bring the two values and the term of 5 years in line?

Formulate this:

The CAGR = \([(FV/PV)^{(1/5)}] - 1
\]

CAGR = \([(55,256/43,295)^{0.20}] - 1
\]

CAGR = \([(1.276)^{0.20}]\]

CAGR = 1.05 - 1 = 5 percent
A Car Loan

You want to purchase a new car. You find the car and negotiate a price, $35,000 all-in. You make a $5,000 down payment and borrow the remaining $30,000 from your credit union at 6% over 4 years. What is a good approximation for your monthly payment?
Car Loan

Let’s find your annual payment – just as we did earlier - and divide by twelve (months).

- You are borrowing a PV = $30,000.
- You will pay-back this $30,000 PV with a 4 year 6% ordinary annuity.
- The PV of your four annual payments must be equal to $30,000. That is …. And this is IMPORTANT.
- You must return the same PV that you borrowed.
Car Loan

• You must return the same PV that you borrowed.
• This means that the annual payments will be somewhat greater than $7,500, which is $30,000 / 4.
• In other words, because you will be paying-off the loan with future dollars, the lender will need more than $30,000 of them.
• If you understand this, corporate finance is in your hands.
Formulation

PVFA’s transform a series of future payments or deposits into a Present Value.

\[
\text{Annuity} \times PVFA(r, t) = \text{Present Value}
\]

\[
\text{Annuity} = \frac{PV}{PVFA(r, t)}
\]

\[
\text{Annuity} = \frac{PV}{\left[1-(1+r)^{-t}\right]/r}
\]

\[
\text{Annual Payment} = \frac{$30,000}{PVFA(6\%, 4)}
\]
Car Loan Example, con’t

Here is the formula we need to solve:

\[
\text{Payment} = \frac{\$30,000}{PVFA(6\%, 4)}
\]

Go to Table 4 and find the Factor at 4 and 6%.

<table>
<thead>
<tr>
<th>Present Value of $1 Annuity Table of Factors</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
</tr>
<tr>
<td>1.00%</td>
</tr>
<tr>
<td>------------------------------------------------</td>
</tr>
<tr>
<td>Periods</td>
</tr>
<tr>
<td>------------------------------------------------</td>
</tr>
<tr>
<td>1     0.9901</td>
</tr>
<tr>
<td>2     1.9704</td>
</tr>
<tr>
<td>3     2.9410</td>
</tr>
<tr>
<td>4     3.9020</td>
</tr>
</tbody>
</table>
Car Loan Example, con’t

Insert the PVFA(6%, 4) = 3.4651 into the formula:
Annual Payment x 3.4652 = $30,000
Annual Payment = $30,000 / 3.4652
$ 8,657.74 = $30,000 / 3.4652

This means that an approximation of your monthly payment is:
$ 8,657.74 / 12 = $ 721.48
Car Loan Example proves that

1) The Loan Payments on the 4 year 6% $30,000 loan are $8,657.74 per year, and

2) $8,657.74 per year are the annual future payments that return a present value of $30,000 @ 6%.

3) Thus, the loan payments are the present value of a $30,000 loan meaning the Lender gets $30,000 of present value from four future payments of $8,658 each.

4) Therefore, if our calculations are correct, the Lender should be indifferent between:
   a) lending to you at 6% for 4 years;
   b) putting the $30,000 in the bank at 6%.

Let’s check this ….
We will compare:
1) the Lender’s Future Value of a single $30,000 deposited for 4 years @6% and
2) the Future Value of $8,658 per year for 4 years @6%.

Bear in mind that someone with $30,000, and a 4 year investment horizon, has two choices:
(a) put it all in the bank earning 6% per year, or
(b) or lend it to you for 4 years expecting 4 annual payments.
Comparing the Lender’s two choices are no different in Future Value:

<table>
<thead>
<tr>
<th></th>
<th>Choice (a)</th>
<th>Choice (b)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>FV Factors</td>
<td>$30,000 in the Bank Compounding</td>
</tr>
<tr>
<td>0</td>
<td>1.0000</td>
<td>30,000</td>
</tr>
<tr>
<td>1</td>
<td>1.0600</td>
<td>31,800</td>
</tr>
<tr>
<td>2</td>
<td>1.1238</td>
<td>33,708</td>
</tr>
<tr>
<td>3</td>
<td>1.1910</td>
<td>35,730</td>
</tr>
<tr>
<td>4</td>
<td>1.2625</td>
<td>37,874</td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

4th payment
3rd payment
2nd payment
1st payment
Future Value of all four Payments